where

$$A_1 = 1 + 2 + 3 + \cdots + p - 1$$
$$A_2 = 1\cdot 2 + 1\cdot 3 + \cdots + 2\cdot 3 + \cdots + (p-2)(p-1)$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$A_{p-1} = 1\cdot 2\cdot 3\cdots(p-1).$$

It is proved by Bachmann that each of these A's, except A_{p-1}, is divisible by p.

Obviously, from the expression of $P(x)$ in factor form $P(p) = (p-1)!$ It follows that $P(p) - (p-1)! = p^{p-1} - A_1p^{p-2} + \cdots + A_{p-3}p^2 - A_{p-2}p = 0$. If then A_{p-3} is divisible by p, A_{p-2} will be divisible by p^2. If $p = 3$, however, $A_{p-3} = 1$, or rather there is no A_{p-3}, and our reasoning breaks down.

Now expand (1). We shall evidently have $P(2p) - (p-1)! = (2p)^{p-1} - A_1(2p)^{p-2} + \cdots + A_{p-3}(2p)^2 - A_{p-2}(2p)$. But A_{p-3} is divisible by p, and A_{p-2} by p^2. Therefore, (1) is divisible by p^3, and $(2p)! - 2\cdot p!p!$ by p^5.

398. Proposed by R. D. CARMICHAEL, Indiana University.

In the equation $x^3 + \alpha x + \beta = 0$, α is an integer divisible by p^2 and β is an integer divisible by p, p being a prime number. Prove that β is divisible by p^3 if the equation is reducible.

SOLUTION BY ELMER SCHUYLER, Brooklyn, New York.

Let $(x^2 + lx + m)(x + n) \equiv x^3 + \alpha x + \beta$. Then since $\alpha = p^2t$ and $\beta = pr$, $l = -n$; $m - n^2 = p^2t$; $mn = pr$. Hence, $m = pr/n$ and $pr/n - n^2 = p^2t$, or $pr - n^3 = p^2nt$. Consequently, n is divisible by p and $n = ps$, since n clearly is an integer. Whence $pr - p^3s^3 = p^3st$ or $r/p^2 = st - s^3$. Hence, we have $\beta = p^3(st - s^3)$.

Also solved by B. LIBBY, G. W. HARTWELL, A. M. HARDING, and ELIJAH SWIFT.

399. Proposed by W. H. BUSSEY, University of Minnesota.

A borrows from B \$1,500 and pays back \$34 a month for 63 months. If the last payment closes the account, what rate of interest has A been paying.

SOLUTION BY GEO. W. HARTWELL, Hamline University.

This problem is equivalent to the following: B pays A \$1,500 for an annuity of \$34 per month to run 63 months. What is the rate realized?

Let $r =$ the annual rate. Then $r/12 =$ the monthly rate. Hence,

$$34\left(1 + \frac{r}{12}\right)^{62} + 34\left(1 + \frac{r}{12}\right)^{61} + \cdots + 34 \equiv 34\left[\frac{\left(1 + \frac{r}{12}\right)^{63} - 1}{\frac{r}{12}}\right] = 1{,}500,$$

or the total number of dollars realized.

Solving this equation by the usual method, we find $r = 14.31$ per cent.

Also solved by G. Y. SASNOW, ALBERT R. NAUER, WILLIAM CULLUM, W. C. EELLS, and CLIFFORD N. MILLS.

400. Proposed by C. N. SCHMALL, New York City.

Sum the series $1 + 2x + 3x^2 + 4x^3 + \cdots$.
(BROMWICH, *Infinite Series*, p. 129, ex. 1.)

I. SOLUTION BY GEO. W. HARTWELL, Hamline University.

Let

$$S = 1 + 2x + 3x^2 + 4x^3 + \cdots.$$

Then

$$(1 - x)S = 1 + x + x^2 + x^3 + x^4 + \cdots = \frac{1}{1 - x}, \text{ for } -1 < x < +1.$$

Hence

$$S = \frac{1}{(1 - x)^2}.$$

II. SOLUTION BY ELIJAH SWIFT, Princeton University.

The series converges for $-1 < x < +1$ and may be integrated in the open interval -1 to $+1$.

Then $\int_0^x S(x)dx = x + x^2 + x^3 + \cdots$, which is equal to $\frac{x}{1 - x}$. Differentiating this, we obtain $\frac{1}{(1 - x)^2}$ as the sum of the given series.

III. SOLUTION BY J. BROOKS SMITH, Hampden Sidney, Va.

The following solution is given by CHARLES SMITH, *Treatise on Algebra*, ex. 1, p. 413.

$$S_{n+1} = 1 + 2x + 3x^2 + 4x^3 + \cdots + (n + 1)x^n,$$

$$(1 - x)^2 = 1 - 2x + x^2.$$

Hence,

$$(1 - x)^2 \cdot S_{n+1} = 1 + x^{n+1}[n - 2(n + 1)] + (n + 1)x^{n+2}$$

(all the other terms vanishing on account of the identity,

$$k - 2(k - 1) + k - 2 \equiv 0).$$

Hence

$$(1 - x)^2 S_{n+1} = 1 - (n + 2)x^{n+1} + (n + 1)x^{n+2};$$

whence

$$S_{n+1} = \frac{1}{(1 - x)^2} - \frac{(n + 2)x^{n+1} - (n + 1)x^{n+2}}{(1 - x)^2}.$$

When $n \doteq \infty$, $S_{n+1} \doteq S \doteq \frac{1}{(1 - x)^2}$ for $-1 < x < +1$.

Solved in various other ways by ELMER SCHUYLER, B. LIBBY, HORACE OLSON, F. M. MORGAN, A. L. MCCARTY, C. HORNUNG, A. M. HARDING, CLIFFORD N. MILLS, H. C. FEEMSTER, OSCAR SCHMIEDEL, and A. G. CARIS.

A solution of 396 was received from ELIJAH SWIFT too late for credit in the last issue.

GEOMETRY.

428. **Proposed by R. D. CARMICHAEL, Indiana University.**

On a given chord of a circle as a base construct a right triangle with vertex outside of the circle such that its hypotenuse shall be bisected by its point of intersection with the circle. Are ruler and compasses sufficient to construct a triangle whose hypotenuse shall be thus divided in any ratio whatever?